Delay Optimal Multichannel Opportunistic Access

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Abstract—The problem of minimizing queueing delay of opportunistic access of multiple continuous time Markov channels is considered. A new access policy based on myopic sensing and adaptive transmission (MS-AT) is proposed. Under the framework of risk sensitive constrained Markov decision process with effective bandwidth as a measure of queueing delay, it is shown that MS-AT achieves simultaneously throughput and delay optimality. It is shown further that both the effective bandwidth and the throughput of MS-AT are two-segment piece-wise linear functions of the collision constraint (maximum allowable conditional collision probability) with the effective bandwidth and throughput coinciding in the regime of tight collision constraints. Analytical and simulations comparisons with the myopic sensing and memoryless transmission (MS-MT) policy which is throughput optimal but delay suboptimal in the regime of tight collision constraints.

Index terms—Delay optimal medium access, effective bandwidth, opportunistic access, and constrained risk sensitive Markov decision process.

I. INTRODUCTION

WE consider in this paper delay optimal policies for a secondary user in a multichannel hierarchical overlay cognitive network [1], where a cognitive user can sense and transmit on one of the N channels assigned to the primary users. A secondary user should only transmit on a channel where the primary user is not transmitting, and its transmissions are subject to interference constraints imposed by primary incumbents.

In such hierarchical cognitive networks, transmission opportunities for a secondary user depend on random traffic patterns of the primary users, which makes it necessary that the secondary user has a queue that holds the arrival packets. Thus packets of a secondary transmitter are subjects to random delays. For applications with delay constraints, there is a need to find a sensing and transmission policies that minimize queueing delays caused by random transmission opportunities and transmission failures due to collisions with the primary users.

Effective bandwidth (or effective capacity in the terminology used in [2]–[4]) is an indirect way to measure the delay performance of a policy for applications either with bursty random arrivals at or random departures from the transmitter queue; the latter may be caused by channel fading as in [2], [3] or random transmission opportunities considered in this paper.

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To characterize the delay due to random arrivals or departures, Kelly argued in [5] that "*constraints on the probability that buffer space or delay exceeds a certain threshold is more important than constraints on mean values.*" The idea is that a provider offering services should guarantee the subscriber a measure of "effective bandwidth" that takes into account randomness either in the arrival or the departure processes. This viewpoint, implicitly espoused earlier in [6], led to the class of approaches based on the theory of large deviations. See [7]–[11] and references therein.

In the context of opportunistic secondary transmissions, the notion of effective bandwidth is defined as follows. Given the buffer size $b \gg 1$ of the secondary transmitter and ϵ, the effective bandwidth is the maximum arrival rate $a(\epsilon, b)$ of the secondary user traffic such that, despite the randomness in transmission opportunities and collisions with primary users, the probability of the queue length exceeding b is capped below ϵ. For applications with constant rate of arrivals, the queue size of a secondary user is proportional to packet delays. Therefore the effective bandwidth can be interpreted as the sustainable arrival rate with delay constraint.

On the surface, transmission delays caused random opportunities resembles that caused by fading, which has been studied in [2]–[4]. There are, however, some important differences. What we are interested in are *delay optimal policies* that choose which channel to sense and the way to transmit; the problem of finding an optimal policy was not the objective of [2]–[4]. In addition, the collision constraints imposed in this paper do not have a correspondence in the problems studied earlier.

A. Summary of Results and Contribution

We consider N identical and independent primary channels, modeled by on-off continuous time Markov processes, with "on" indicating the channel is being used by the primary user and "off" a transmission opportunity for the secondary user. The secondary user adopts a slotted sensing-before-transmission protocol which is defined by a sensing policy that determines which channel to sense and a transmission policy that specifies the probability of transmission.

If the secondary user transmits on a particular channel, a collision with the primary user is possible because, as the licensed incumbent, the primary user transmits whenever it has a packet. We assume that the secondary user abides by the interference constraints imposed by the primary users. Here the interference to a primary user is measured by the collision probability between the primary and secondary users conditioned on the event that the primary user transmits. In this paper, the set of admissible policies includes those whose

interference to primary channels is uniformly bounded above by γ.

The main results of this paper are as follows. First, we propose a simple policy referred to as MS-AT (myopic sensing and adaptive transmission). The sensing policy is the myopic policy originally proposed by Zhao, Krishnamachari, and Liu [12] for slotted primary user systems. The key idea of MS-AT is an adaptive transmission policy that adjust adaptively its transmission probabilities based on feedbacks of past transmissions (ACK/NAK). In particular, given the maximum acceptable collision constraint γ, the adaptive transmission policy sets a target rate $\tau(\gamma)$ of successful transmissions and adjust its transmission probabilities based on the estimated rate of successful transmissions. We note that the proposed MS-AT policy can be interpreted as a debt-based policy whose principle is used to establish feasible throughput optimality in the deadline scheduling problem [13]–[15], though the deadline scheduling problem and the debt-based policy considered in [13]–[15] are very different from those in this paper.

Second, we establish that MS-AT simultaneously achieves delay and throughput optimality under any collision constraint. In establishing the optimality of MS-AT, the main idea is to formulate the problem of maximizing effective bandwidth subject to collision constraints as one of risk sensitive Markov decision process which adopts an exponential objective function (instead of the usual linear objective function in the risk neutral Markov decision process). In particular, in the regime of loose collision constraints, the optimality of MS-AT is shown by the technique of Hernandez and Marcus who established the multiplicative dynamic programming representation of the unconstrained risk sensitive Markov decision process in [16].

Third, we provide simple characterizations of the throughput and effective bandwidth as functions of the collision constraint parameter γ. It is shown that both the effective bandwidth and the throughput of MS-AT are two-segment piece-wise linear functions of γ, and the effective bandwidth and throughput coincide in the regime of tight collision constraints as illustrated in Fig. 1.

The result that the effective bandwidth matches to the throughput in the regime of tight collision constraints seems surprising since the effective bandwidth is typically strictly smaller than the throughput [17]. A commonly accepted intuition is that the randomness in the input process requires more bandwidth, and randomness in the output process decreases the sustainable rate traffic if the delay is required to be small. This intuition is consistent with our result when the collision constraints are loose as it can be observed in Section IV that under loose collision constraints the optimal effective bandwidth is strictly smaller than the optimal throughput. However, when the collision constraint is tight, the delay optimal policy offers effective bandwidth equal to the optimal throughput (see Theorem 1). The key insight, as presented in Section III, is that the secondary user can make its packet departure process favorable by an adaptive transmission policy.
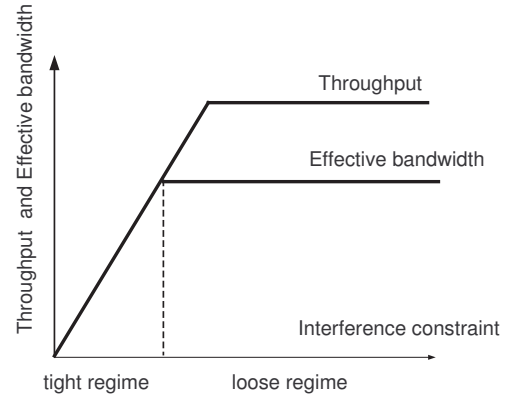

Fig. 1. Illustration of throughput and effective bandwidth of MS-AT as piece-wise linear functions of collision constraints.

B. Related Work

Substantial amount of work exists for opportunistic access with throughput as the performance measure. In [18], Zhao *et. al.* consider the case when primary users follow a slotted Markov transmission structure under the per slot interference constraint. The myopic sensing policy is shown to be throughput optimal for identical, positively correlated Markovian channels [12], [19]. The continuous time Markovian occupancy model has also been treated using the throughput measure (see, *e.g.*, [20]–[22]). Adopting a periodic channel sensing policy, the optimal transmission policy is obtained in the framework of constrained Markov decision process under the average interference constraint [22], which further leads to the throughput optimality of the periodic sensing with memoryless transmission (PS-MT) policy when the collision constraints are tight[1] [23], [24]. The multiuser maximum throughput region of opportunistic access is studied in [25] in which the throughput optimal policies employ memoryless transmission policy.

The use of effective bandwidth to analyze delay for opportunistic spectrum access was first considered in [26] where the authors obtain in closed form the effective bandwidth of the myopic sensing policy with slotted primary transmission, without establishing the optimality of myopic sensing. In [27] both throughput and effective bandwidth are considered for multiuser cognitive access. The effective bandwidth optimality of myopic sensing for two identical Markov channels under loose collision constraints is obtained in [27], whereas our result in this paper establishes the optimality for a general number of homogeneous channels under loose collision constraint as well as new optimal policy for general collision constraint. In [28] the authors propose an ALOHA based policy for multiuser opportunistic access and analyze its delay performance via a fluid model.

The optimization of effective bandwidth under tight collision constraint can be formulated as constrained risk sensitive

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[1]the tight collision constraint in [23], [24] is different from the tight collision regime for MS-MT

Markov decision. The only existing results to our knowledge are [29], [30], where [29] considers the set of Markov policies whereas [30] treats the general constrained Markov decision process with the objective function and the constraints given by general utility functions and establishes the existence of an optimal policy in the set of general history dependent policies without any structural property or computation procedure of the optimal policy. In this paper, we exploit the special form of the utility function and the associated linear constraints, which leads to a structured optimal policy.

II. NETWORK MODEL

The network model and the assumptions are described as follows. The hierarchical overlay cognitive network consists of N parallel primary channels indexed by $1, \ldots, N$ and a secondary user. Each primary user transmits on its dedicated channel, the occupancy of which is assumed to be independent, identically distributed continuous time Markov process. The state space of the channel is $\{1(\text{busy}), 0(\text{idle})\}$ and the holding times are exponentially distributed with mean μ^{-1} and λ^{-1} for busy and idle states, respectively. The secondary user opportunistically accesses the primary channels in a slotted sensing-before-transmission manner with slot length T. In each time slot the secondary user senses one out of the N channels (sensing is assumed to be perfect) and decides whether or not to transmit in the sensed channel. Successful transmission of c bits is achieved in slot t if the secondary user transmits in a channel that is idle throughout slot t. In particular, if the secondary user senses a channel to be busy in slot t, no transmission will be attempted in this slot since no successful transmission but collision will be achieved. We aim to maximize the effective bandwidth subject to the collision constraints imposed by the primary users.

A. Effective Bandwidth

Effective bandwidth characterizes the quantity of service available to the secondary user via the opportunistic channel with QoS constraint. To give the definition of the effective bandwidth we consider the queueing process of the secondary user.

Assume that the incoming traffic of the secondary user is a constant arrival process with intensity of a bits per slot, and the bits are stored in a buffer of size $b \gg 1$ before being transmitted. For a fixed opportunistic access policy π, denote by Q_t^π the queue length at the end of slot t. Then $(Q_t^\pi)_{t \geq 0}$ satisfies the following recursion

$$Q_t^\pi = \max\{Q_{t-1}^\pi + a - R_t^\pi, 0\}, \quad t \geq 1, \quad Q_0^\pi = 0.$$

where $(R_t^\pi)_{t \geq 0}$ is the output process of the opportunistic channels ($R_t^\pi = c$ if successful transmission is achieved in slot t, and $R_t^\pi = 0$ otherwise). Under regularity conditions, Glynn and Whitt show in [8] that, if the policy π is such that the queue is stable, then Q_n^π converges in distribution to a steady state distribution Q_∞^π with the decay rate

$$\lim_{x \to \infty} \frac{1}{x} \log \Pr(Q_\infty^\pi > x) = -\theta^\pi(a), \qquad (1)$$

where $\theta^\pi(a)$ is the unique positive solution of the equation

$$a\theta + \Psi_R^\pi(-\theta) = 0, \qquad (2)$$

and $\Psi_R^\pi(\theta)$ is the Gartner-Ellis limit

$$\Psi_R^\pi(\theta) = \lim_{n \to \infty} \frac{1}{n} \log \mathbb{E}_\pi[\exp(\theta \sum_{k=1}^n R_k^\pi)].$$

The decay rate $\theta^\pi(a)$ in Eq. (1) depends on the secondary user's traffic arrival rate a and the output process $(R_t^\pi)_{t \geq 0}$; the latter depends on the primary channel statistics and the opportunistic access policy. With the decay rate in Eq. (1) the packet drop (buffer overflow) probability can be approximated by

$$\Pr[Q_\infty^\pi > b] \approx \exp(-\theta^\pi(a)b), \qquad (3)$$

for large buffer size b.

With the constraint that the buffer overflow probability is capped by ϵ and adopting the large buffer approximation in Eq. (3), the effective bandwidth $a^\pi(\epsilon, b)$ can be defined as

$$a^\pi(\epsilon, b) \triangleq \max\{a : \exp(-b\theta^\pi(a)) \leq \epsilon\}. \qquad (4)$$

It can then be shown (see [26]) that the following effective bandwidth formula holds.

$$a^\pi(\epsilon, b) = \lim_{n \to \infty} \frac{\log \mathbb{E}_\pi \exp(\frac{\log(\epsilon)}{b} \sum_{t=1}^n R_t^\pi)}{n \frac{\log(\epsilon)}{b}}. \qquad (5)$$

The above expression shows a key connection between effective bandwidth and throughput. Applying the Jensen's inequality, we have $a^\pi(\epsilon, b) \leq \lim_{n \to \infty} \frac{1}{n} \mathbb{E}_\pi \sum_{t=1}^n R_t^\pi$. In general, the effective bandwidth is strictly smaller than the throughput, except in certain special scenarios (see Section III). Asymptotically as the buffer size approaches infinity ($b \to \infty$) or the QoS requirement is relaxed ($\epsilon \to 1$), the effective bandwidth does converge to the throughput.

B. Collision Constraints

The transmissions of the secondary user are subject to collision constraints imposed by the primary users. The collision is measured in a long term average manner. Specifically, for each primary user i the scaled infinite horizon average collision with primary user i must be bounded by γ; the scale infinite horizon average collision for primary user i is

$$C_{\pi,i} = \frac{1}{1 - v(0) \exp(-\lambda T)} \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \sum_{t=1}^n \chi_{i,t},$$

where $\chi_{i,t}$ is the indicator of collision with primary user i in slot t, and the scale uses the reciprocal of $1 - v(0) \exp(-\lambda T)$ ($v(0) = \mu/(\lambda + \mu)$ is the stationary distribution of idle state), i.e., the steady state probability of primary user i transmitting in a certain slot. Given the collision parameter γ, the set of admissible policies $\Pi(\gamma)$ is given by the set of policies that meet the interference constraints, i.e., $\Pi(\gamma) = \{\pi : C_{\pi,i} \leq \gamma, \forall i\}$. We aim to characterize the maximum effective bandwidth for the admissible policy set $\Pi(\gamma)$.
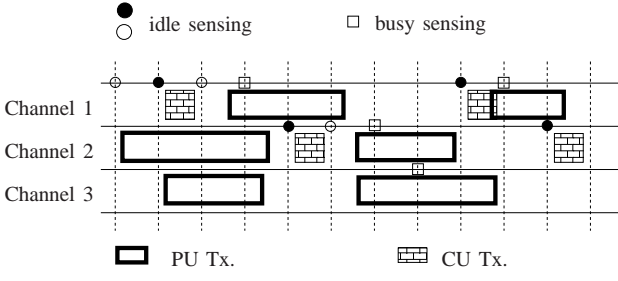
Fig. 2. Illustration of MS-MT policy. Filled(Open) circle: secondary user decides to(not to) transmit.

C. Opportunistic Access Policy

The opportunistic access policy consists of two components: the channel sensing policy which selects a channel to sense in each slot based on the past history, and the transmission policy which specifies the transmission probability upon idle sensing results. (Upon busy sensing results the secondary user will keep silent.)

We first describe the myopic sensing policy (MS). In [12], [19], myopic sensing is shown to be throughput optimal for independent, identically distributed, positively correlated, discrete time Markov channels with a simple round-robin structure. Specifically, the secondary user first fixes an ordered list of the N channels. To start, the secondary user senses the first channel in the list and keeps sensing it until the first busy sensing result. Then the secondary user switches to the next channel in the list and keeps sensing this channel. The secondary user cycles through the channel list in this stay-when-idle, switch-when-busy manner.

When it comes to the transmission policy, the transmission probability is determined, in general, based on the entire history. A simple (but in general suboptimal) transmission policy is the memoryless transmission (MT) policies where the transmission decision depends only on the current sensing outcome with constant probability of transmission upon idle sensing results. We consider in this paper a new class of transmission policy referred to as adaptive transmission (AT) policies. As discussed in detail in Section III, the transmission strategies at time slot t depends on the history of transmissions in the past.

In this paper, we consider two types of opportunistic access policies: MS-MT—the myopic sensing and memoryless transmission [25], [27] (with constant transmission probability μ_{MS} chosen to satisfy the interference constraints) and a new policy referred to as MS-AT—myopic sensing and adaptive transmission. Figs. 2 illustrates the sample path of the MS-MT policy and the MS-AT policy is described in Section III.

III. OPTIMALITY OF MS-AT

First define $\theta = \log(\epsilon)/b$ to be the effective bandwidth parameter for buffer size b and packet drop probability ϵ. Rewrite the effective bandwidth formula (5) with θ

$$a^\pi(\epsilon, b) = \lim_{n \to \infty} \frac{\log \mathbb{E}_\pi \exp(\theta \sum_{t=1}^n R_t^\pi)}{n\theta}.$$

A. Adaptive Transmission Policy

According to the effective bandwidth formula (5) a simple upper bound can be easily obtained, *i.e.*, the optimal throughput under collision parameter γ. To achieve this upper bound, we first describe the mechanism of adaptive transmission policy (AT). We employ the acknowledgement of the secondary user to aid the transmission decision. Specifically, after sensing a channel to be idle in slot t the secondary user counts the total number of acknowledgements received up to slot $t-1$, denoted by A_t and transmits with probability 1 if $A_t < \tau t$. Otherwise if $A_t \geq \tau t$, the secondary user stays silent. Here τ is the parameter of AT policy and controls the collision caused by the secondary user.

The adaptive transmission policy can be also thought of as a debt based transmission policy. The target of the secondary user is set to be transmitting τt packets within the first t slots, thus if $A_t < \tau t$, the secondary user is in debt and thus needs to transmits with probability 1, and otherwise, if $A_t \geq \tau t$, the secondary user has a reasonable balance and stays silent.

The optimality of the MS-AT policy with respect to effective bandwidth and the optimal effective bandwidth as a function of the collision parameter γ is provided in Theorem 1.

Theorem 1: For any collision parameter γ, there exists τ such that the MS-AT policy with parameter τ is feasible and optimal for the constrained effective bandwidth optimization with collision parameter γ. The parameter τ of the optimal MS-AT policy is given by

$$\tau = \frac{N\gamma \exp(-\lambda T)(1 - v(0)\exp(-\lambda T))}{1 - \exp(-\lambda T)}, \quad (6)$$

where $v(0) = \mu/(\lambda + \mu)$ is the stationary distribution of idle state. The optimal effective bandwidth is given by

$$\text{EB}^* = \min\{\frac{N\gamma \exp(-\lambda T)(1 - v(0)\exp(-\lambda T))}{1 - \exp(-\lambda T)}, \frac{\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})}{\theta}\}, \quad (7)$$

where

$$\tilde{\theta} = \log(1 - \exp(-\lambda T)(1 - \exp(c\theta))), \quad (8)$$

$$\Psi_X^{\text{MS}(\infty)}(\tilde{\theta}) = \lim_{n \to \infty} \frac{1}{n} \log \mathbb{E}^{\text{MS}(\infty)}[\exp(\tilde{\theta} \sum_{k=1}^n R_k^{\text{MS}(\infty)})],$$

and $\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$ is the optimal effective bandwidth when the collision parameter γ is loose (the minimum in Eq. (7) is assumed by the second term).

Also the MS-AT policy with parameter τ is optimal with respect to the throughput, with optimal throughput

$$\text{TH}^* = \min\{\frac{N\gamma \exp(-\lambda T)(1 - v(0)\exp(-\lambda T))}{1 - \exp(-\lambda T)},$$
$$\lim_{n \to \infty} \frac{1}{n} \mathbb{E}^{\text{MS}(\infty)} \sum_{t=1}^n R_t^{\text{MS}(\infty)}\}.$$

Theorem 1 provides the solution to the constrained effective bandwidth optimization problem by establishing the optimality of MS-AT for all collision parameter γ. To prove Theorem 1, we need to first prove a list of lemmas. In the later presentation we assume $c = 1$ without loss of generality.

B. MS-AT with $\tau = \infty$

We first establish the effective bandwidth optimality of MS-AT policy with $\tau = \infty$ under loose collision parameter γ. We denote the MS-AT policy with $\tau = \infty$ by MS-AT(∞). By the structure of MS-AT, the secondary user always transmits upon idle sensing results under MS-AT(∞) policy. Denote by $X_t^{\text{MS}(\infty)}$ the indicator of the secondary user having an idle sensing result in slot t, following MS-AT(∞) policy.

Call a collision parameter γ loose if the MS-AT(∞) policy is admissible with respect to γ. We are interested in proving that the MS-AT(∞) policy has better effective bandwidth than any other admissible opportunistic access policy. To this end, we relax the collision constraint and turn to show that the MS-AT(∞) policy has better effective bandwidth than any other opportunistic access policy, whether admissible or not. This relaxation leads to an unconstrained risk sensitive Markov decision process, the optimality of which is provided in Lemma 1.

Lemma 1: Under loose collision constraint, MS-AT(∞) is effective bandwidth optimal over all admissible policies in $\Pi(\gamma)$.

Proof: In view of Eq. (5) and the fact that the effective bandwidth parameter $\theta = \log(\epsilon)/b < 0$, to show the effective bandwidth optimality we just need to show that MS-AT(∞) minimizes $M_{R,n}^{\pi}(\theta) = \mathbb{E}_{\pi} \exp(\theta \sum_{t=1}^{n} R_t^{\pi})$ over all $\pi \in \Pi(\gamma)$ for all $n \geq 1$. We then make connection between $M_{R,n}^{\pi}(\theta)$ and $M_{X,n}^{\pi}(\theta) = \mathbb{E}_{\pi} \exp(\theta \sum_{t=1}^{n} X_t^{\pi})$, where X_t^{π} is the indicator of the secondary user having an idle sensing result in slot t under opportunistic access policy π. Specifically, we have the following inequality

$$
\begin{aligned}
M_{R,n}^{\pi}(\theta) &= \mathbb{E}[\mathbb{E}[e^{\theta \sum_{t=1}^{n} R_t^{\pi}} \mid X_1^{\pi}, \ldots, X_n^{\pi}]] \\
&= \mathbb{E}[\prod_{t=1}^{n} \mathbb{E}[e^{\theta R_t^{\pi}} \mid X_t^{\pi}]] \\
&= \mathbb{E}[\prod_{t=1, X_t^{\pi}=1}^{n} [1 - \mu_t^{\pi} e^{-\lambda T}(1 - e^{c\theta})]] \\
&\geq \mathbb{E}[[1 - (\sup_{t=1}^{\infty} \mu_t^{\pi}) e^{-\lambda T}(1 - e^{c\theta})]^{\sum_{t=1}^{n} X_t^{\pi}}] \quad (9) \\
&= M_{X,n}^{\pi}(\log(1 - (\sup_{t=1}^{\infty} \mu_t^{\pi}) e^{-\lambda T}(1 - e^{c\theta}))) \quad (10)
\end{aligned}
$$

where μ_t^{π} is the transmission probability used by policy π in slot t.

Under loose collision constraint it holds for opportunistic access policy π that $\mu_t^{\pi} \leq 1$. On the other hand, the MS-AT(∞) policy always transmit with probability 1. Therefore equality holds in Eq. (9) for MS-AT(∞). Thus we just need to show that for an arbitrary opportunistic access policy π,

$$
M_{X,n}^{\pi}(\theta) \geq M_{X,n}^{\text{MS}(\infty)}(\theta), \quad (11)
$$

for all $\theta < 0$.

We prove Eq. (11) with the following multiplicative dynamic programming recursion [16] for the finite horizon problem with $\mathbb{E}e^{\theta \sum_{k=1}^{K} X_k}$ as the objective function. First

define

$$
\begin{aligned}
V_K(\boldsymbol{\omega}) &= \min_{1 \leq a \leq N} \mathbb{E}e^{\theta X_a(\boldsymbol{\omega})} \\
V_t(\boldsymbol{\omega}) &= \min_{1 \leq a \leq N} \mathbb{E}[e^{\theta X_a(\boldsymbol{\omega})} V_{t+1}(\mathcal{T}(\boldsymbol{\omega}, a))]
\end{aligned}
$$

for $t = 1, \ldots, K-1$, where a is the dummy variable denoting the current action at time t of choosing to sense the ath channel, and $V_t(\cdot)$ is the value function, defined as the minimum expected future objective function that can be achieved starting from t when the information state is $\boldsymbol{\omega}$, *i.e.*, $V_t(\boldsymbol{\omega}) = \inf_{\pi} \mathbb{E}_{\pi}[\exp(\theta \sum_{k=t}^{K} X_k) \mid \boldsymbol{\omega}]$, where the information state $\boldsymbol{\omega}$ is the length N vector with the ith component being the conditional probability that the ith primary channel will be sensed idle in the next slot (see [19] for the definition and interpretation of the channel information state vector $\boldsymbol{\omega}$). Expanding the dynamic programming recursion based on the sensing outcome, we have

$$
\begin{aligned}
V_K(\boldsymbol{\omega}) &= \min_{1 \leq a \leq N} \mathbb{E}e^{\theta X_a(\boldsymbol{\omega})} = \min_{1 \leq a \leq N} \{\omega_a e^{\theta} + (1 - \omega_a)\} \\
V_t(\boldsymbol{\omega}) &= \min_{1 \leq a \leq N} \mathbb{E}[e^{\theta X_a(\boldsymbol{\omega})} V_{t+1}(\mathcal{T}(\boldsymbol{\omega}, a))] \\
&= \min_{1 \leq a \leq N} \{\omega_a e^{\theta} V_{t+1}(\mathcal{T}(\boldsymbol{\omega}, a \mid 0)) \\
&\quad + (1 - \omega_a) V_{t+1}(\mathcal{T}(\boldsymbol{\omega}, a) \mid 1)\}
\end{aligned}
$$

We can proceed from here and show that picking the largest component in the channel information state vector $\boldsymbol{\omega}$ to sense solves the multiplicative dynamic programming equation, and thus prove the effective bandwidth optimality of MS-MT, following the line of lemmas in [19]. One key lemma in [19] (Lemma 3) needs to be modified to the multiplicative version

$$
e^{\theta} V_{t+1}(\omega_1, \ldots, \omega_{n-2}, p_{11}, p_{01}) \geq V_{t+1}(p_{01}, \omega_1, \ldots, \omega_{n-2}, p_{11}).
$$

We omit the details of the proof due to limited space. ∎

Note that under loose collision constraints as γ increases the optimal effective bandwidth stays constant; the optimal value is

$$
\text{EB}^*(\epsilon, b) = \frac{\Psi_R^{\text{MS}(\infty)}(\theta)}{\theta} = \frac{\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})}{\theta},
$$

where $\theta = \log(\epsilon)/b$, and

$$
\tilde{\theta} = \log(1 - \exp(-\lambda T)(1 - \exp(c\theta))), \quad (12)
$$

and the last equality follows from Eq. (10).

C. Structural Properties of MS-AT

We analyze the structure of the adaptive transmission policy (AT) coupled with myopic channel sensing policy. Specifically, there will be consecutive time slots in which the secondary user is in good balance (therefore stays silent) and is in debt (therefore keeps transmitting upon idle sensing results), respectively. We denote the consecutive intervals by $I_1, \ldots, I_n, \ldots$ and $B_1, \ldots, B_n, \ldots$. We also denote by $i_1, \ldots, i_n, \ldots$ and $b_1, \ldots, b_n, \ldots$ the first time slot in the intervals $I_1, \ldots, I_n, \ldots$ and $B_1, \ldots, B_n, \ldots$. Note that in the first slot it always holds that $A_1 = 0 < \tau$, which indicates that the interval B_1 (B_n) comes before interval I_1 (I_n).

Lemma 2: As τ increases, the effective bandwidth associated with MS-AT with parameter τ is non-decreasing.

Proof: Prove this lemma with a sample path argument. Specifically, for arbitrary $\tau_1 < \tau_2$ and any horizon n we show that along any sample path of the channel state realization the MS-AT policy with τ_2 accumulates more successful transmissions than the MS-AT policy with τ_1, *i.e.*,

$$\sum_{t=1}^{n} r_t^{\tau_1} \leq \sum_{t=1}^{n} r_t^{\tau_2}, \tag{13}$$

where the lower case r indicates the quantities are conditioned on a sample path realization. Eq. (13) will follow if one can show for any integer k that the MS-AT policy with τ_2 achieves the kth successful transmission no later than the MS-AT policy with τ_1 for the specific channel state realization.

Assume otherwise, *i.e*, for some integer k the kth successful transmission occurs earlier under the MS-AT policy with τ_1. Without loss of generality assume that the k is the smallest among the integers that satisfy this condition. Thus a contradiction can be easily drawn since in the time slot of the kth successful transmission under the MS-AT policy with τ_1, the MS-AT policy with τ_2 has more debt than the MS-AT policy with τ_1, and thus will decide to transmit, which is guaranteed to be successful since the MS-AT policy with τ_1 has the kth successful transmission in that very time slot. Therefore the contradiction proves Eq. (13), and the claim of the lemma follows. ∎

Lemma 3: For all $k \geq 1$ it holds that

$$A_{i_k} \geq i_k \tau, \ A_{b_k} < b_k \tau, \tag{14}$$

and

$$A_{i_k - 1} < (i_k - 1)\tau, \ A_{b_k - 1} \geq (b_k - 1)\tau. \tag{15}$$

Proof: We prove the inequalities using the alternating nature of the sequence of intervals we define and the structure of the MS-AT policy.

By the structure of the adaptive transmission policy, in time slot t if the MS-AT policy attempts to transmit (*i.e.*, $t \in B_k$ for some k), then $A_t < \tau t$. Similarly, if the MS-AT policy stays silent (*i.e.*, $t \in I_k$ for some k), then $A_t \geq \tau t$.

For all $k \geq 1$, by definition i_k is the first time slot in the kth idle period, therefore $A_{i_k} \geq i_k \tau$. For all $k \geq 1$, by definition $i_k - 1$ is the last time slot before the kth idle period, therefore $i_k - 1$ lies in a certain busy interval, which indicates $A_{i_k-1} < (i_k - 1)\tau$. The other two inequalities can be shown similarly, except that there is one special case for $k = 1$ ($b_1 - 1 = 0$ does not lie in any idle or busy interval). To accommodate this special case we define by convention $A_0 = 0$. ∎

Lemma 4: For any $\tau < \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$,

1) the lengths of the intervals I_k are bounded by $\frac{1}{\tau}$ almost surely,
2) the lengths of the intervals B_k have finite expectation.

Proof: In time slot 1, the secondary user is always transmitting since $A_1 = 0 < \tau$. Therefore the kth idle interval I_k is ahead of the kth busy interval B_k. Thus the idle interval

I_k starts from slot i_k (inclusion) and ends in slot $b_{k+1} - 1$ (inclusion). Therefore the length of the idle interval I_k is $|I_k| = b_{k+1} - i_k$.

According to the structure of MS-AT policy and Lemma 3, it holds that

$$A_{i_k} \leq A_{i_k - 1} + c < (i_k - 1)\tau + c,$$

$$A_{b_{k+1}} \geq A_{b_{k+1} - 1} \geq (b_{k+1} - 1)\tau,$$

and because during the idle interval I_k nothing is transmitted, it follows that $A_{b_{k+1}} = A_{i_k}$. Therefore

$$(b_{k+1} - 1)\tau < (i_k - 1)\tau + c,$$

which indicates

$$|I_k| = b_{k+1} - i_k < \frac{c}{\tau} = \frac{1}{\tau}.$$

In other words, idle periods cannot be longer than $1/\tau$ since every $1/\tau$ slots there will be $\tau \cdot \frac{1}{\tau} = 1$ packet due for transmission based on the debt based interpretation of the adaptive transmission policy.

The busy interval B_k begins from slot b_k (inclusion) and ends in slot $i_k - 1$ (inclusion). According to Lemma 3 it holds that $A_{b_k} < \tau b_k$ and

$$A_{b_k} \geq A_{b_k - 1} \geq \tau b_k - \tau. \tag{16}$$

This can be interpreted as follows. In time slot b_k the MS-AT policy has accumulated $A_{b_k} \in [\tau b_k - \tau, \tau b_k)$ successful transmissions, which is still short compared with the target τb_k.

By definition slot i_k is the first slot after slot b_k in which the MS-AT policy has no debt. Again according to Lemma 3 it holds that $A_{i_k} \geq \tau i_k$ and

$$A_{i_k} \leq A_{i_k - 1} + c < \tau(i_k - 1) + c, \tag{17}$$

i.e., the balance in slot i_k is less than $c - \tau$.

Now consider the event $\{|B_k| = n\}$. It holds that

$$\begin{aligned} \{|B_k| = n\} &= \{i_k = n + b_k\} \\ &\subset \{A_{n+b_k} < \tau(n + b_k - 1) + c\} \quad (18) \\ &\subset \{A_{n+b_k} - A_{b_k} < \tau n + c\} \quad (19) \\ &= \{\frac{A_{n+b_k} - A_{b_k}}{n} < \tau + \frac{c}{n}\}, \quad (20) \end{aligned}$$

where Eq. (18) and Eq. (19) make use of Eq. (16) and Eq. (17), respectively.

By definition of busy interval the secondary user is always transmitting upon idle sensing results in the interval $[b_k, n-1+b_k]$. Therefore as n tends to infinity the following convergence holds exponentially fast since the joint state which combines the channel state vector and the current channel index the secondary user is at evolves as a finite state recurrent Markov chain (see [25] Section IV.B),

$$\lim_{n\to\infty} \frac{1}{n} \mathbb{E}^{\text{MS}(\tau)}(A_{n+b_k} - A_{b_k}) = \lim_{n\to\infty} \frac{1}{n} \mathbb{E}^{\text{MS}(\infty)} \sum_{t=1}^{n} R_t^{\text{MS}(\infty)}$$

$$> \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta,$$

where the middle term is the optimal throughput of the MS-AT policy with $\tau = \infty$, *i.e.*, the secondary user always transmits upon idle sensing results.

Since $\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta > \tau$, for n large enough $\tau + c/n$ will lie to the left of $\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$, and thus to the left of $\lim_{n\to\infty} \frac{1}{n}\mathbb{E}^{\text{MS}(\infty)} \sum_{t=1}^{n} R_t^{\text{MS}(\infty)}$ with gap at least $\lim_{n\to\infty} \frac{1}{n}\mathbb{E}^{\text{MS}(\infty)} \sum_{t=1}^{n} R_t^{\text{MS}(\infty)} - \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$.

Therefore the set in Eq. (20) corresponds to a deviation of at least $\lim_{n\to\infty} \frac{1}{n}\mathbb{E}^{\text{MS}(\infty)} \sum_{t=1}^{n} R_t^{\text{MS}(\infty)} - \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$ and has exponentially decaying probability in n, and so does the set $\{|B_k| = n\}$, which indicates that the lengths of the intervals B_k have finite expectation. ■

Lemma 5: For any $\tau < \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$, we have for the MS-AT policy with $\tau = \infty$ (MS(∞) policy)

$$\sup_n M_{R,n}^{\text{MS}(\infty)}(\theta) e^{-\theta\tau n} < \infty, \qquad (21)$$

where $M_{R,n}^{\text{MS}(\infty)}(\theta) = \mathbb{E}^{\text{MS}(\infty)} \exp(\theta \sum_{t=1}^{n} R_t^{\text{MS}(\infty)})$.

Proof: Following Eq. (10) one can easily obtain that

$$\lim_{n\to\infty} \frac{1}{n} \log M_{R,n}^{\text{MS}(\infty)}(\theta) = \Psi_X^{\text{MS}(\infty)}(\tilde{\theta}). \qquad (22)$$

Therefore for any $\tau < \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$, there exists $\epsilon > 0$ such that $\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta - \epsilon - \tau > 0$. By the limit in Eq. (22), we have that there exists N such that for all $n > N$

$$\frac{1}{n\theta} \log M_{R,n}^{\text{MS}(\infty)}(\theta) > \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta - \epsilon, \qquad (23)$$

which implies

$$M_{R,n}^{\text{MS}(\infty)}(\theta) < e^{n\theta(\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta-\epsilon)}, \qquad (24)$$

and further for all $n > N$

$$M_{R,n}^{\text{MS}(\infty)}(\theta) e^{-\theta\tau n} < e^{n\theta(\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta-\epsilon-\tau)} \leq e^{N\theta(\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta-\epsilon-\tau)}. \qquad (25)$$

Therefore we conclude

$$\sup_n M_{R,n}^{\text{MS}(\infty)}(\theta) e^{-\theta\tau n} \leq \max\{e^{N\theta(\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta-\epsilon-\tau)},$$
$$\max_{1\leq i\leq N}\{M_{R,i}^{\text{MS}(\infty)}(\theta) e^{-\theta\tau i}\}\} < \infty,$$

thus proving the lemma. ■

Lemma 6: For any $\tau < \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$, the effective bandwidth of the MS-AT policy with parameter τ is no less than τ.

Proof: By the definition of effective bandwidth we have the effective bandwidth of the secondary user is given by

$$a^{\text{MS}(\tau)}(\epsilon, b) = \lim_{n\to\infty} \frac{\log \mathbb{E}^{\text{MS}(\tau)} \exp(\theta \sum_{t=1}^{n} R_t^{\text{MS}(\tau)})}{n\theta}. \qquad (26)$$

for buffer size $b \gg 1$ and $\theta = \log(\epsilon)/b$.

Define $T(n)$ to be the slot index that corresponds to the last slot before n (inclusion) that is the start of an interval, *i.e.*,

$$T(n) = \max\{i \leq n : i = i_k \text{ or } i = b_k \text{ for some } k\}. \qquad (27)$$

Then the analysis deal with two possibilities, $T(n) = i_k$ for some k and $T(n) = b_k$ for some k.

1) $T(n) = i_k$ for some k. Then we have

$$\mathbb{E}e^{\theta(\sum_{t=1}^{n} R_t - \tau n)}$$
$$= \mathbb{E}e^{\theta(A_{T(n)} - \tau T(n) + \sum_{k=T(n)}^{n} R_k - \tau(n-T(n)))}$$
$$\leq \mathbb{E}e^{\theta(\sum_{k=T(n)}^{n} R_k - \tau(n-T(n)))}$$
$$\leq \mathbb{E}e^{-\tau|I_k|\theta}$$
$$\leq \mathbb{E}e^{-\theta}$$

where we used the inequalities $A_{i_k} \geq \tau i_k$ and $|I_k| < 1/\tau$. Therefore we have

$$\lim_{n\to\infty} \frac{1}{n} \log \mathbb{E}e^{\theta(\sum_{t=1}^{n} R_t - \tau n)} \leq 0,$$

which leads to

$$\lim_{n\to\infty} \frac{1}{n\theta} \log \mathbb{E}e^{\theta \sum_{t=1}^{n} R_t} \geq \tau.$$

2) $T(n) = b_k$ for some k. Then we have

$$\mathbb{E}e^{\theta(\sum_{t=1}^{n} R_t - \tau n)}$$
$$= \mathbb{E}e^{\theta(A_{T(n)} - \tau T(n) + \sum_{k=T(n)}^{n} R_k - \tau(n-T(n)))}$$
$$\leq \mathbb{E}e^{\theta(-\tau + \sum_{k=T(n)}^{n} R_k - \tau(n-T(n)))}$$
$$\leq e^{-\theta\tau} \sup_k M_{R,k}(\theta) e^{-\theta\tau k}$$

where we used the inequality $A_{b_k} - \tau b_k \geq -\tau$ and Lemma 5.

Therefore we have

$$\lim_{n\to\infty} \frac{1}{n} \log \mathbb{E}e^{\theta(\sum_{t=1}^{n} R_t - \tau n)} \leq 0,$$

which also leads to

$$\lim_{n\to\infty} \frac{1}{n\theta} \log \mathbb{E}e^{\theta \sum_{t=1}^{n} R_t} \geq \tau.$$

Therefore we have proved that the effective bandwidth of the MS-AT policy is at least as large as the parameter τ. ■

Lemma 7: The throughput of the MS-AT policy with parameter τ is at most τ.

Proof: The throughput of the secondary user is given by $\lim_{n\to\infty} \frac{1}{n}\mathbb{E}^{\text{MS}(\tau)} \sum_{t=1}^{n} R_t^{\text{MS}(\tau)}$. Again split the analysis into two possibilities, $T(n) = i_k$ for some k and $T(n) = b_k$ for some k.

1) $T(n) = i_k$ for some k. Then we have

$$\mathbb{E}(\sum_{t=1}^{n} R_t - \tau n)$$
$$= \mathbb{E}(A_{T(n)} - \tau T(n) + \sum_{k=T(n)}^{n} R_k - \tau(n - T(n)))$$
$$\leq \mathbb{E}(c - \tau), \qquad (28)$$

where Eq. (28) is due to the fact that from slot $T(n)$ to slot n the secondary user stays silent. Thus it holds that

$$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}(\sum_{t=1}^{n} R_t - \tau n) \leq 0,$$

which leads to

$$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} R_t \leq \tau.$$

2) $T(n) = b_k$ for some k. Then we have

$$\mathbb{E}(\sum_{t=1}^{n} R_t - \tau n)$$

$$= \mathbb{E}(A_{T(n)} - \tau T(n) + \sum_{k=T(n)}^{n} R_k - \tau(n - T(n)))$$

$$\leq \mathbb{E}\sum_{k=T(n)}^{n} c$$

$$\leq c\mathbb{E}|B_k|.$$

Due to the fact that the expected length of a busy period is finite, we have in parallel with the previous case

$$\lim_{n\to\infty} \frac{1}{n}\mathbb{E}(\sum_{t=1}^{n} R_t - \tau n) \leq 0,$$

which leads to

$$\lim_{n\to\infty} \frac{1}{n\theta}\mathbb{E}\sum_{t=1}^{n} R_t \leq \tau,$$

Therefore we have proved that the throughput of the MS-AT policy is at most as large as the parameter τ. ∎

Corollary 1: For any $\tau < \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$, the effective bandwidth and throughput of MS-AT with parameter τ is both τ.

Proof: Application of Jensen's inequality implies that for the same opportunistic access policy π the effective bandwidth is no greater than the throughput $\lim_{n\to\infty} \frac{1}{n}\mathbb{E}_\pi \sum_{t=1}^{n} R_t^\pi$. However, we have already established that the throughput of MS-AT is no greater than the parameter τ and the effective bandwidth is no less than the parameter τ. Therefore combining the two facts, we conclude that the throughput and the effective bandwidth are both equal to the parameter τ. ∎

Lemma 8: For any collision parameter γ, the following choice of parameter τ

$$\tau = \frac{N\gamma \exp(-\lambda T)(1 - v(0)\exp(-\lambda T))}{1 - \exp(-\lambda T)} \quad (29)$$

is feasible.

Proof: The MS-AT policy only transmits upon idle sensing results. Therefore for any horizon n the ratio between the expected number of total successful transmissions and the expected number of total collisions is $\frac{1-\exp(-\lambda T)}{\exp(-\lambda T)}$. With MS-AT policy the total number of successful transmissions the secondary user achieves during the first t slots satisfies

$$A_{t+1} < \tau t + c. \quad (30)$$

Taking the expectation of Eq. (30) implies that the expected number of total successful transmissions is upper bounded by $\tau t + c$ and thus the expected number of total collisions is bounded by $\frac{(\tau t+c)(1-\exp(-\lambda T))}{\exp(-\lambda T)}$. Then we divide this upper bound expression by t, take the limit $t \to \infty$ and take into

account the scaling coefficient $1 - v(0)\exp(-\lambda T)$ (see Section II-B). Now we obtain that the sum of the collision $C_{\text{MS}(\tau),i} \leq N\gamma$.

Note that the channel sensing policy and the transmission policy is symmetric with respect to the N channels. Therefore the lemma is proved. ∎

Then we are ready to prove Theorem 1.

Proof: Note that the parameter $\tau(\gamma)$ given by Eq. (29) is no less than the best throughput for any opportunistic access policy under collision limit γ (see [25]).

We also know from Corollary 1 that for any $\tau < \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$ the effective bandwidth and throughput of MS-AT with parameter τ is both τ. Therefore for any $\tau < \Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$ the effective bandwidth of MS-AT with parameter τ matches with the optimal throughput upper bound, thus optimal with respect to effective bandwidth.

Combining this fact with Lemma 2 and the fact that MS-AT(∞) has optimal effective bandwidth $\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})/\theta$ for loose collision parameters, we conclude that for any collision parameter γ the parameter τ given by

$$\tau = \frac{N\gamma \exp(-\lambda T)(1 - v(0)\exp(-\lambda T))}{1 - \exp(-\lambda T)}$$

achieves the optimal effective bandwidth

$$\text{EB}^* = \min\{\frac{N\gamma \exp(-\lambda T)(1 - v(0)\exp(-\lambda T))}{1 - \exp(-\lambda T)}, \frac{\Psi_X^{\text{MS}(\infty)}(\tilde{\theta})}{\theta}\}.$$

The throughput optimality of the MS-AT policy can be proved similarly. ∎

IV. SIMULATION RESULTS

We plot the throughput and effective bandwidth versus the collision parameter γ for MS-AT. In comparison, we also plot the throughput and effective bandwidth versus the collision parameter γ for MS-MT, the policy with myopic sensing but memoryless transmission. In the simulation the number of primary channel is set to be $N = 2$. The slot length in the simulation is taken to be $T = 0.25$ms. The channel parameters we use are $\mu = 1/2\text{ms}^{-1}$ and $\lambda = 1/3\text{ms}^{-1}$. The effective bandwidth parameter $\theta = \log(\epsilon)/b$ is taken to be -0.08.

Fig. 3 depicts the throughput and effective bandwidth versus the collision parameter γ for the MS-AT and MS-MT policies. The plot shows that the effective bandwidth and throughput matches up to a certain point for MS-AT policy, and then levels up at optimal effective bandwidth for loose collision parameter, validating the optimality result for MS-AT policy. On the other hand, the throughput and effective bandwidth for the MS-MT policy deviates in the entire γ-axis, showing that the MS-MT policy is indeed suboptimal when the collision constraint is tight. On the other hand, when the collision constraint is loose, the two policies both give the optimal effective bandwidth. The optimal effective bandwidth deviates from the throughput in the loose collision regime, as predicted in the presentation. Also note that in Fig. 3 the optimal throughput as well as the effective bandwidth is first linearly increasing in γ and then levels up.
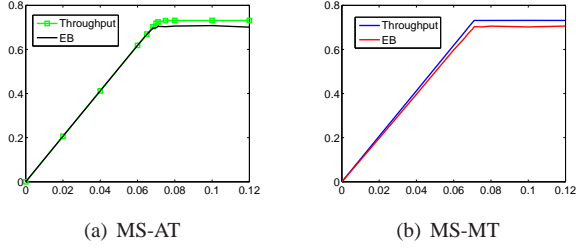
(a) MS-AT

(b) MS-MT

Fig. 3.   Throughput and Effective bandwidth of MS-AT and MS-MT.
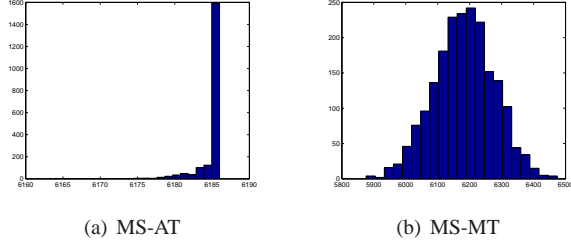

(a) MS-AT

(b) MS-MT

Fig. 4.   Empirical distribution of MS-AT and MS-MT.

The empirical distribution of A_t is plotted in Fig. 4. It can be observed that under MS-AT policy, the distribution of A_t is more condensed near the target τt, whereas under MS-MT policy, the distribution of A_t is more far apart from τt.

V. CONCLUSIONS

Multichannel opportunistic access of homogeneous continuous time Markov channels is considered. The objective is to design an opportunistic access policy that maximizes the effective bandwidth under the interference constraints. This paper shows that a myopic sensing plus adaptive transmission policy is optimal with respect to both effective bandwidth and throughput under all levels of collision constraint. The optimality result may find applications in cognitive radio networks for spectrum overlay with QoS requirement. Although we focus on single user scenario in this paper, the technique can be extended to multiuser scenario and similar results can be established for the multiuser effective bandwidth region.

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